AP Calculus AB
Review 25, No Calculator Permitted on MC

Complete all the following on notebook paper.

_____ 1.

If $f(x) = x^2 e^x$, then the graph of f is decreasing for all x such that

(A) $x < -2$ (B) $-2 < x < 0$ (C) $x > -2$ (D) $x < 0$ (E) $x > 0$

_____ 2.

If $y = \arctan(e^{2x})$, then $\frac{dy}{dx} =$

(A) $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\frac{2e^{2x}}{1+e^{4x}}$ (C) $\frac{e^{2x}}{1+e^{4x}}$ (D) $\frac{1}{\sqrt{1-e^{4x}}}$ (E) $\frac{1}{1+e^{4x}}$

_____ 3.

What is the volume of the solid generated by rotating about the x-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \frac{\pi}{3}$?

(A) $\frac{\pi}{\sqrt{3}}$

(B) π

(C) $\pi\sqrt{3}$

(D) $\frac{8\pi}{3}$

(E) $\pi \ln\left(\frac{1}{2} + \sqrt{3}\right)$

_____ 4.

Which of the following is equal to $\int_0^{\pi} \sin x \, dx$?

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$ (B) $\int_0^{\pi} \cos x \, dx$ (C) $\int_{-\pi}^{0} \sin x \, dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx$ (E) $\int_{\pi}^{2\pi} \sin x \, dx$

_____ 5.

Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\frac{30}{\pi^2}$ cm (E) $\frac{10}{\pi}$ cm

_____ 6.

If $f(x)=\begin{cases} x & \text{for } x \le 1 \\ \frac{1}{x} & \text{for } x > 1, \end{cases}$ then $\int_0^e f(x)dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) e (E) $e+\frac{1}{2}$

_____ 7.

If $\frac{dy}{dx}=\frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1,4]$ is

(A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2

_____ 8.

If f is continuous on the interval $[a,b]$, then there exists c such that $a<c<b$ and $\int_a^b f(x)dx =$

(A) $\frac{f(c)}{b-a}$ (B) $\frac{f(b)-f(a)}{b-a}$ (C) $f(b)-f(a)$ (D) $f'(c)(b-a)$ (E) $f(c)(b-a)$

_____ 9.

$\int_0^1 x\left(x^2+2\right)^2 dx =$

(A) $\frac{19}{2}$ (B) $\frac{19}{3}$ (C) $\frac{9}{2}$ (D) $\frac{19}{6}$ (E) $\frac{1}{6}$

_____ 10.

If $f(x) = \ln\left(\sqrt{x}\right)$, then $f''(x) =$

(A) $-\frac{2}{x^2}$ (B) $-\frac{1}{2x^2}$ (C) $-\frac{1}{2x}$ (D) $-\frac{1}{2x^{\frac{3}{2}}}$ (E) $\frac{2}{x^2}$

11. –AB5B (No Calculator)

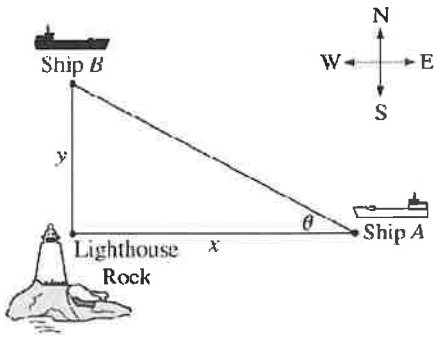
Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

12. –AB6B (No Calculator)

Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.



(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.